Analysis of Gas-Lubricated Bearings with a Coupled Boundary Effect for Micro Gas Turbine ©

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Due to the limitation of micro-machining technology, the bearing aspect ratio for a micro gas turbine is limited to a small value. Therefore, the load-carrying capacity of such bearing is inherently small. Thus, it is desirable to predict the load-carrying capacity as accurately as possible without being unnecessarily conservative. The direct numerical analysis of bearing characteristics was performed by mass conservation at coupled boundary. Journal bearing was numerically simulated for two cases - 1) an externally pressurized bearing with four feeding holes and 2) a hydrodynamic bearing. Characteristics such as pressure distribution at the bearing surface, load-carrying capacity as a function of aspect ratio with a coupled boundary effects are investigated for micro gas turbine bearings. It is shown that a model of thrust and journal bearings coupled at boundary generally predicts a greater load-carrying capacity than uncoupled models.

KEY WORDS
Gas-Lubricated Bearings; Load-Carrying Capacity; Coupled Boundary

INTRODUCTION
Recently, developments of micro-machining technology have led to widespread studies in miniaturization of conventional sys-

NOMENCLATURE

\[ A = \text{reference area of bearing, m}^2; A = \pi \cdot D \cdot L \text{ or } \pi(D^2-d^2)/4 \]
\[ A_0 = \text{reference curtain area of feeding hole, m}^2; A_0 = 2\pi \cdot r_s \cdot h_0 \]
\[ C = \text{radial clearance, m} \]
\[ C_d = \text{discharge coefficient} \]
\[ D = \text{diameter of journal and outer diameter of thrust bearing, m} \]
\[ d = \text{inner diameter of thrust bearing, m} \]
\[ e = \text{eccentricity, m} \]
\[ h = \text{film thickness, m}; H_j = h/C, H_T = h/r_i \]
\[ h_0 = \text{reference film thickness of thrust bearing, m} \]
\[ L = \text{length of journal bearing, m} \]
\[ \dot{m} = \text{mass flow rate, kg/s} \]
\[ p, P = \text{pressure, Pa}; P = p/p_a \]
\[ p_a = \text{ambient pressure, Pa} \]
\[ P_c, P_j = \text{supply pressure, Pa}; P_c/P_j \]
\[ p_0, P_0 = \text{pressure at coupled boundary, Pa}; P_0 = p_0/p_a \]
\[ R = \text{gas constant, J/kg K} \]
\[ r_i = \text{radius of feeding hole, m} \]
\[ T = \text{absolute temperature, K} \]
\[ w, W, \Delta W = \text{load-carrying capacity, N}; W = w/p_a \cdot A, \Delta W = (W_c - W_{UC})/W_{UC} \]

\[ \Gamma_s = \text{dimensionless parameter}, \Gamma_s = \left[ \frac{12\mu \cdot C_d \cdot A \cdot \sqrt{RF}}{p_a r_i} \right] \]
\[ \Lambda, \Delta = \text{bearing number, } \Lambda = \frac{6p_0}{\rho a}, \Delta = \frac{\Lambda}{\Lambda} \]
\[ \mu = \text{viscosity of gas, Ns/m}^2 \]
\[ \epsilon = \text{eccentricity ratio, } \epsilon = e/C \]
\[ \kappa = \text{adiabatic number} \]
\[ \omega = \text{angular velocity, rad/s} \]
\[ \theta = \text{angular coordinates of journal and thrust bearings} \]
\[ \zeta = \text{longitudinal coordinates of journal bearing} \]
\[ \eta = \text{radial coordinates of thrust bearing} \]
\[ \hat{i}, \hat{j}, \hat{k} = \text{unit vector on } \theta \text{ axis} \]
\[ \hat{i}, \hat{j}, \hat{k} = \text{unit vector on } \zeta \text{ axis} \]
\[ \hat{i}, \hat{j}, \hat{k} = \text{unit vector on } \eta \text{ axis} \]

SUBSCRIPT

\[ i = \text{arbitrary grid point number on } \theta \text{ axis} \]
\[ j = \text{arbitrary grid point number on } \zeta \text{ axis} \]
\[ k = \text{arbitrary grid point number on } \eta \text{ axis} \]
\[ C = \text{coupled boundary} \]
\[ UC = \text{uncoupled boundary} \]
\[ J = \text{journal bearing} \]
\[ T = \text{thrust bearing} \]
systems. Especially, MEMS-based power sources (power MEMS), including micro generators, micro fuel cells, micro thrusters and micro gas turbines have been studied very actively. These power MEMS are aiming to achieve very high energy density by using chemical reaction energy. Generally, it is known that micro combustion engines, such as a micro gas turbine, have 10 times energy density that available from the best existing batteries (Ian, Gautam and Yang-Shen (1998)).

However, micro gas turbine depends on rotating machinery. This micro-rotating machinery requires extremely high rotation rates of more than a million rpm to achieve sufficient efficiency due to their millimeter-scale diameters. At this condition, bearing elements are the major obstacles of successful development. Because of their small scale, it is difficult to use a typical bearing, such as rolling element or oil lubricated bearing, for a micro gas turbine. Naturally, a need of a new type of bearing has been grown. One of possible candidate solutions is a gas-lubricated bearing.

In fact, a gas-lubricated bearing has been used widely in conventional rotating systems due to their distinct advantages compared to rolling element or oil lubricated bearings. First, there is no defilement due to leakage of lubricant such as oil. Second, it is possible to achieve an extremely high speed because friction force is very small. These advantages could be extended into micro-rotating machinery, and with these advantages, the studies to develop a gas-lubricated bearing that could be used in micro rotating machinery are performing widely. (Piekos and Breuer (1999)) studied the stability of a gas-lubricated journal bearing in micro gas turbine using pseudo-spectral orbit simulation, and (Orr (2000)) studied the stability of a gas-lubricated journal bearing with very low aspect ratio of L/D=0.075 using Galerkin method. It should be noted that the journal bearing has an extremely small load-carrying capacity, it could be a means of overcoming a defect of extremely short bearings in terms of load-carrying capacity. Thus, in this paper, the behavior of coupled bearing in a micro gas turbine is investigated.

**ANALYSIS**

**Analysis Model**

The analysis was performed based on a real micro gas turbine test rig. Figure 1 shows a cut-away view of rotor and housing of the test rig. The bearing consists of three pieces, an upper thrust pad, a journal pad and a lower thrust pad. Both the upper and lower thrust pads have four feeding holes with a radius of 50 µm. The feeding holes of the upper and lower thrust pads are aligned. For the journal bearing, two types of pads were manufactured. One has four feeding holes (the externally pressurized journal bearing case) and the other has no feeding holes (the hydrodynamic journal bearing case).

It should be noted that the journal bearing has an extremely low L/D ratio about 0.083 owing to the limitations of micro-machining technology.

**Reynolds Equation**

Although high rotation rates of micro gas turbine, flow in bearing clearance is in laminar region due to small diameter of rotor and bearing clearance. Therefore, it is relevant to use Reynolds equation as a governing equation. Using the conventional assumptions of laminar, compressible, and isoviscous flow, the non-dimensional Reynolds equations are written as follows:

\[
\frac{\partial}{\partial \theta} \left( \rho H \frac{\partial P}{\partial \theta} \right) + \frac{\partial}{\partial \eta} \left( \rho H \frac{\partial P}{\partial \eta} \right) = \Lambda \frac{\partial (PH)}{\partial \theta} .
\]

\[
\frac{\partial}{\partial \eta} \left( \eta \rho H \frac{\partial P}{\partial \eta} \right) + \frac{1}{\eta} \frac{\partial}{\partial \theta} \left( \rho H \frac{\partial P}{\partial \theta} \right) = \Lambda \frac{\partial (PH)}{\partial \theta} .
\]
Numerical Analysis

Equations [1] and [2] are nonlinear partial differential and can be written in vector form as follows;

\[
\nabla \cdot (PHV, P - \Delta (PH)) = 0 \tag{3}
\]

\[
\nabla \cdot \left( (PH \frac{\partial P}{\partial \eta}) \hat{k} - \left( \frac{PH}{\eta} \frac{\partial P}{\partial \theta} - \Lambda \eta PH \right) \hat{\tau} \right) = 0 \tag{4}
\]

To apply the finite difference method (FDM), grids are generated to be of 121, 29 and 37 along the \( \zeta \), \( \xi \) and \( \eta \) directions, respectively. Equations [3] and [4] are integrated over the control surface for arbitrary grid points.

\[
\int_{S_{i,j}} (PHV, P - \Delta (PH)) dS_{i,j} = 0 \tag{5}
\]

\[
\int_{S_{i,k}} \left( (PH \frac{\partial P}{\partial \eta}) \hat{k} - \left( \frac{PH}{\eta} \frac{\partial P}{\partial \theta} - \Lambda \eta PH \right) \hat{\tau} \right) dS_{i,k} = 0 \tag{6}
\]

By applying Gauss' divergence theorem, these integrated Eqs. [5] and [6] are transformed from surface integrals to line integrals as follows;

\[
\int_{L_{i,j}} (PHV, P - \Delta (PH)) \cdot \hat{n} dl = 0 \tag{7}
\]

\[
\int_{L_{i,k}} \left( (PH \frac{\partial P}{\partial \eta}) \hat{k} - \left( \frac{PH}{\eta} \frac{\partial P}{\partial \theta} - \Lambda \eta PH \right) \hat{\tau} \right) \cdot \hat{n} dl = 0 \tag{8}
\]

Integrating Eqs. [7] and [8] along four grid lines at arbitrary grid point \((i,j)\) or \((i,k)\), discrete algebraic equations at each grid point are obtained. Finally, by using central difference formula, these equations are approximated as follows;

\[
\begin{align*}
(\bar{A}_{i} + \bar{A}_{j} + \bar{A}_{k}) P_{i,j} = & \bar{A}_{i} P_{i+1,j} + \bar{A}_{j} P_{i,j+1} + \bar{A}_{k} P_{i,j-1} - \bar{A}_{i} + \bar{A}_{j} + Q_{i,j}, \tag{9}
\end{align*}
\]

\[
\begin{align*}
\bar{P}_{i,j} = & \frac{H_{i,j}^{(i,k)}}{\Delta \xi} \left[ \frac{H_{i,j}^{(i,k)}}{\Delta \eta} P_{i,j+1} + \frac{H_{i,j}^{(i,k)}}{\Delta \xi} P_{i,j-1} \right] + \frac{H_{i+1,j}^{(i,k)}}{\Delta \xi} P_{i+1,j} + \frac{H_{i,j+1}^{(i,k)}}{\Delta \eta} P_{i,j+1} - \bar{A}_{i} + \bar{A}_{j} + Q_{i,j}. \tag{10}
\end{align*}
\]

The coefficients, A and B, are given in Eqs. [A1]-[A12] in the appendix. The supply flow rate \( Q_{s} \) (Eqs. [A13] and [A14] in the appendix) at the feeding hole located at the grid point is determined by the isentropic condition of compressible fluid, while it is zero at the other grid points. Equations [9] and [10] are solved iteratively. The accuracy of solution is examined by comparing inlet flow rate from feeding holes and outlet flow rate from a bearing boundary.

Coupled Boundary

The continuity condition of mass flow rate at the common control surface is applied to couple the thrust and journal bearings. That is, mass flow rate must be equivalent across a common control surface. Figure 2 provides the schematic of common control surface. Before applying the continuity condition, two assumptions are made. First, the mass flow along the \( \zeta \) and \( \eta \) directions is conserved at the coupled end. Second, there is no pressure drop at the coupled end. The first assumption implies that any mass flow along \( \zeta \) and \( \eta \) direction does not dissipate in the \( \theta \) direction. This assumption is valid when the L/D ratio is less than 0.5 (Hamrock (1994)). The second assumption means that the pressure at the journal bearing coupled boundary equals to the pressure at the thrust bearing coupled boundary. From these assumptions, following relation is obtained.

\[
\bar{m}_{\zeta} + \bar{m}_{\eta} = 0 \tag{11}
\]

These mass flow rates are given by

\[
\bar{m}_{\zeta} = -\rho \frac{H_{i}^{(i,k)} \frac{\partial P_{i,j}}{\partial \zeta}}{12 \mu \frac{\partial \zeta}{\partial \xi}} = \frac{P_{i,j}^{(i,k)} \frac{\partial P_{i,j}}{\partial \zeta}}{12 \mu RT \frac{\partial \zeta}{\partial \xi}} \tag{12}
\]

\[
\bar{m}_{\eta} = -\rho \frac{H_{i}^{(i,j)} \frac{\partial P_{i,j}}{\partial \eta}}{12 \mu \frac{\partial \eta}{\partial \xi}} = \frac{P_{i,j}^{(i,j)} \frac{\partial P_{i,j}}{\partial \eta}}{12 \mu RT \frac{\partial \eta}{\partial \xi}} \tag{13}
\]

Substituting Eqs. [12] and [13] into Eq. [11], the following relation is obtained;

\[
H_{i}^{(i,j)} \frac{\partial P_{i,j}}{\partial \eta} + H_{j}^{(i,j)} \frac{\partial P_{i,j}}{\partial \zeta} = 0 \tag{14}
\]

Applying the central difference formula for Eq. [14] (Eqs. [A15] and [A16] of appendix) and using the second assumption, \( P_{i,j}^{(i,jend)} = P_{i,j}^{(i,jend)} = P_{\rho} \). Eq. [14] becomes an algebraic equation as follows;

\[
\bar{P}_{i,j} = \frac{1}{H_{i}^{(i,j)} / \Delta \eta} \left[ \frac{H_{i}^{(i,k)} / \Delta \eta}{\Delta \xi} P_{i,j+1} + \frac{H_{i+1,j}^{(i,k)}}{\Delta \xi} P_{i+1,j} + \frac{H_{i,j+1}^{(i,k)}}{\Delta \eta} P_{i,j+1} - \bar{A}_{i} + \bar{A}_{j} + Q_{i,j} \right] \tag{15}
\]

Equation [15] determines the pressure at the coupled boundary.
of thrust and journal bearings, and Eqs. [9] and [10] determine the pressure within the boundary.

RESULTS AND DISCUSSION

Pressure Distribution

Figure 3 shows the difference between the pressure distribution of the uncoupled and coupled cases. The uncoupled case neglects the interaction between the thrust and journal bearings, and the coupled case considers this interaction. When the journal and thrust bearings are uncoupled, the pressure at the boundary equals the ambient pressure. In this case, pressure is not well distributed in the circumferential direction, but only distributed in the longitudinal direction along the feeding holes. This phenomenon occurs because the flow induced by pressure, (i.e. Poiseuille flow), in longitudinal direction is much larger than that in circumferential direction. This means that pressure distribution in the circumferential direction is less significant than that in the longitudinal direction. However, in this case, the load-carrying capacity of bearing is very small due to its low L/D(=0.083) ratio. In the coupled case, an interesting phenomenon occurs. Figure 3(b) shows that the pressure on the bearing surface increases notably. If the interaction between the thrust and journal bearing is considered, the gas flows from the journal to the thrust or from the thrust to the journal bearing. Consequently, this interchange of gas increases the gas flow rate, which increases the pressure on the bearing surface. Clearly, when interaction is considered, a greater load-carrying capacity of bearing is found.

Load-Carrying Capacity

Figure 4 shows a comparison of coupled and uncoupled models. The dimensionless load-carrying capacity of the journal bearing, W_J, is plotted over a range of bearing numbers, Λ, for two aspect ratios (L/D equals 0.083 and 0.5). Figure 4(a) shows the hydrodynamic journal bearing case and Fig. 4(b) shows the externally pressurized journal bearing case.

In the hydrodynamic journal bearing case, the coupled model gives a greater load-carrying capacity than the uncoupled model for both aspect ratios over the whole range of bearing numbers. In the uncoupled case with extremely short width journal bearing (L/D=0.083), the load-carrying capacity of the journal bearing
increases slightly as the bearing number becomes larger. However, the load-carrying capacities in this case are very small for the whole range of bearing numbers. In the coupled case, the load-carrying capacity of the journal bearing increase rapidly as bearing number increases. For the higher aspect ratio (L/D=0.5), a similar trend is observed. However, in this case, the slope of the load-carrying capacity curve decreases as the bearing number increases. In order to compare the influence of coupling on the load-carrying capacity, load-carrying capacity increasing ratio, \(\Delta W_J\), is defined as

\[\Delta W_J = \frac{W_{J,C} - W_{J,UC}}{W_{J,UC}}\]

For L/D=0.5, \(\Delta W_J\) varies from about 0.45 to about 0.52. For L/D=0.083, \(\Delta W_J\) varies from about 3.75 to about 4.50. This means that the load-carrying capacity of the journal bearing as predicted by the coupled model is about four times greater than the capacity predicted by the uncoupled model.

In the externally pressurized journal bearing case, the load-carrying capacity of journal bearing predicted by the coupled model is also generally greater than that predicted by the uncoupled model except at bearing numbers below approximately 2. In this low bearing number region, pressure from the feeding holes is initially generated. At this condition, if the coupling of thrust and journal bearing is considered, the pressure at the diverging wedge, initially a low-pressure region, is increased more than that at the converging wedge, initially a high-pressure region. Because the load-carrying capacity of the journal bearing is the sum of pressure along the circumferential surface of the journal bearing, this reduces the load-carrying capacity of journal bearing. However this phenomenon disappears as bearing number increases. The load-carrying capacity increasing ratio, \(\Delta W_J\), varies from about -0.50 to about 0.50 for L/D=0.5 and from about -0.08 to about 2.50 for L/D=0.083.

By comparing \(\Delta W_J\) of the hydrodynamic and externally pressurized journal bearing cases, it is seen that the coupled model gives a higher estimate of the load-carrying capacity than the uncoupled model for most bearing number, and it is more effective for the low L/D ratio.

Figure 5 gives the dimensionless load-carrying capacity vs. supply pressure curves for each L/D ratio. In the hydrodynamic journal bearing case at constant bearing number and eccentricity ratio, the dimensionless load-carrying capacity does not vary while changes in supply pressure. As the supply pressure increases, the incoming mass flow rate from the thrust bearing to the journal bearing increases accordingly across the whole journal bearing clearance. As a result, dimensionless load-carrying capacity maintains the constant value. However, even it remains the constant value as changing in supply pressure, the coupling of the thrust and journal bearings increase the dimensionless load-carrying capacity. In the externally pressurized journal bearing case, different trends are observed. The dimensionless load-carrying capacity is increasing with supply pressure for both the uncoupled and coupled cases for both aspect ratios because the mass flow rate also increases. However, the trend of the uncoupled and coupled cases are considerable. For L/D equals to 0.5, the dimensionless load-carrying capacity predicted by the coupled model is greater than predicted by the uncoupled model if supply pressure is less than about 2.5 Pa. But if supply pressure is more than about 2.5 Pa, this situation is reversed. This phenomenon is also observed for L/D=0.083 case. This means that, in the range of low bearing number, the coupling does not give a higher estimate of the load-carrying capacity at the high supply pressure region for the externally pressurized journal bearing.

Figure 6 provides the bearing number vs. load-carrying capacity increasing ratio, \(\Delta W_J\), curves for L/D=0.083 and eccentricity ratio=0.5 while changes in supply pressure, \(P_s\). For the hydrodynamic journal bearing case, Fig. 6(a), the load-carrying capacity increasing ratio reduces as the bearing number increases. However, its value is about 400%. The influence of supply pressure of the thrust bearing is not significant. While the supply pressure of thrust bearing changes in 2.0-4.0, the variation of the load-carrying capacity increasing ratio lies within 10%. For the externally pressurized journal bearing case, Fig. 6(b), the load-carrying capacity increasing ratio increases as the bearing number becomes larger. The load-carrying capacity increasing ratio has the negative value for small bearing number and supply pressure. This
means that the coupling of thrust and journal bearing is not always effective to increase the load-carrying capacity. However, for large bearing number region, the coupling increases the load-carrying capacity. By comparing the load-carrying capacity increasing ratio of Figs. 6(a) and 6(b), it is seen that the coupling is more effective in the hydrodynamic journal bearing than in externally pressurized journal bearing. Consequently, the coupling of thrust and journal bearing is very effective way to increase a load-carrying capacity for an extremely short journal bearing and could be a possible design concept of gas-lubricated bearing which is used in micro rotating machinery such as micro gas turbine.

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**REFERENCES**


**APPENDIX**

**Coefficients**

\[ A_i = \frac{P_{st,ij}}{\Delta \theta_{st}} \left( H_{ij, st, j+1/2, j} \cdot \Delta \theta_{st} + H_{ij, st, j+1/2, j} \cdot \Delta \zeta_{st} \right) \]  
\[ A_j = \frac{P_{st,ij}}{\Delta \theta_{ij}} \left( H_{ij, st, j+1/2, j} \cdot \Delta \theta_{ij} + H_{ij, st, j+1/2, j} \cdot \Delta \zeta_{ij} \right) \]  
\[ A_k = \frac{P_{st,ij}}{\Delta \zeta_{ij}} \left( H_{ij, st, j+1/2, j} \cdot \Delta \theta_{ij} + H_{ij, st, j+1/2, j} \cdot \Delta \zeta_{ij} \right) \]  
\[ A_e = \frac{\Lambda \cdot P_{st,ij}}{2} \left( H_{ij, st, j+1/2, j} \cdot \Delta \zeta_{st} + H_{ij, st, j+1/2, j} \cdot \Delta \zeta_{st} \right) \]  
\[ A_f = \frac{\Lambda \cdot P_{st,ij}}{2} \left( H_{ij, st, j+1/2, j} \cdot \Delta \zeta_{ij} + H_{ij, st, j+1/2, j} \cdot \Delta \zeta_{ij} \right) \]  
\[ B_i = \frac{P_{st,ij} H_{ij, st, j+1/2, j} \left( \Delta \theta_{ij} + \Delta \theta_{st} \right)}{\eta \Delta \theta_{st}} \]  

**Fig. 6**—Load-carrying capacity increasing ratio of journal bearing. vs. bearing number, L/D=0.083, ε = 0.5.  
(a) hydrodynamic journal bearing  
(b) externally pressurized journal bearing
\[ B_i = \frac{P_{\alpha_s+t/2} H_{x_{i_2}+t/2} \left( \frac{\Delta \theta_i + \Delta \theta_s}{2} \right) \left( \eta_i + \Delta \eta_{s,i} \right)}{\Delta \eta_{s,i}} \] \tag{A8}

\[ B_i = \frac{P_{\alpha_s+t/2} H_{x_{i_2}+t/2} \left( \frac{\Delta \theta_i + \Delta \theta_s}{2} \right)}{\eta_i \Delta \theta_i} \] \tag{A9}

\[ B_i = \frac{P_{\alpha_s+t/2} H_{x_{i_2}+t/2} \left( \Delta \theta_i + \Delta \theta_s \right)}{2} \left( \eta_i + \Delta \eta_{s,i} \right) \] \tag{A10}

\[ B_i = \Delta \eta_i H_{x_{i_2}+t/2} P_{\alpha_s+t/2} \left( \frac{\Delta \eta_i + \Delta \eta_{s,i}}{2} \right) \] \tag{A11}

\[ B_i = \Delta \eta_i H_{x_{i_2}+t/2} P_{\alpha_s+t/2} \left( \frac{\Delta \eta_i + \Delta \eta_{s,i}}{2} \right) \] \tag{A12}

**Supply Flow Rate**

\[ Q_i = \Gamma_i \cdot P_2 \cdot H \left( \frac{2k}{\kappa + 1} \right) \left( \frac{2}{\kappa + 1} \right)^{\frac{1}{\kappa + 1}} \left( \frac{P}{P_2} \right) \left( \frac{2}{\kappa + 1} \right)^{\frac{1}{\kappa + 1}} \] \tag{A13}

\[ Q_i = \Gamma_i \cdot P_2 \cdot H \left( \frac{2k}{\kappa + 1} \right) \left( \frac{2}{\kappa + 1} \right)^{\frac{1}{\kappa + 1}} \left( \frac{P}{P_2} \right) \] \tag{A14}

**Pressure Gradients**

\[ \frac{\partial P_1}{\partial \eta} = \frac{P_{\left(i.k_{\text{end}}\right)} - P_{\left(i,k_{\text{end}} - 1\right)}}{\Delta \eta_i} \] \tag{A15}

\[ \frac{\partial P_i}{\partial \zeta} = \frac{P_{\left(i,k_{\text{end}}\right)} - P_{\left(i,k_{\text{end}} - 1\right)}}{\Delta \zeta_i} \] \tag{A16}