Torque Performance of a Brushless DC Motor due to Coil Winding Patterns

Gunhee Jang

(Received September, 28, 1995)

Abstract

The winding pattern determines the torque performance of brushless dc motors as well as permanent magnets. This paper analyzes the torque performance of the BLDC motor due to the different winding patterns. The torque was calculated by the Maxwell stress tensor method, integrating the shear component of magnetic traction along the air gap assuming a quasistatic magnetic field, its characteristics were discussed using spectral analysis. Several different cases are investigated, e.g. the reluctance torque, and commutating torque under center tapped Y-winding, Y-winding and Δ-winding. Y-winding proves to be the most efficient one, but it has narrow commutating range and high torque ripple.

Nomenclature

\[ \begin{align*}
    B & : \text{magnetic flux density} \\
    H_s & : \text{source magnetic field intensity} \\
    L_2 & : \text{the period corresponding to rotating angle of the rotor} \\
    L_3 & : \text{the period corresponding to the torque} \\
    T & : \text{torque} \\
    \sigma_i & : \text{stress} \\
    w & : \text{rotor speed} \\
    \mu & : \text{permeability} \\
    Y_{km}, Z_n & : \text{complex Fourier coefficient} \\
    H & : \text{magnetic field intensity} \\
    L_4 & : \text{the period corresponding to tooth angle} \\
    R & : \text{distance from the torque center} \\
    f_i & : \text{traction in i direction} \\
    \delta & : \text{Kronecker delta} \\
    \theta & : \text{angle} \\
    F & : \text{scalar potential} \\
    Y_n & : \text{Fourier phase angle}
\end{align*} \]

1. Introduction

Analysis of torque is important to determine the performance of output of a motor. New high energy materials, such as neodymiumironboron, increases not only the overall torque but also the torque ripple and the reluctance torque, or "cogging". Permanent magnet brushless DC
motors represent the fastest growing segment of the world electric motor market.

These motors are popular because their response is linear and thus can be easily controlled over a fairly wide range of speeds. Operation of these motors at relatively low speeds and loads, however, is difficult due to the existence of reluctance torque in this range. Reluctance torque will also present itself when the motor is "coasting" with no power applied, as is done when the motor speed is controlled by an "on-off" type controller. This type of controller is extremely popular due to its simplicity and low cost. Even at the high speeds and loads, the commutating torque diagram is sinusoidal in shape and produces high torque ripple, defined by the difference between the minimum and maximum torque. The reluctance torque and the commutating torque, proportional to the magnetic flux density squared, between a moving magnet and its surrounding structure creates an instability in that structure. This instability, usually ignored in low flux density devices, limits the design in many high flux machines.

Two traditional methods exist for the calculation of the torque of motor - the virtual work method and the Maxwell stress tensor method. In the virtual work method, the torque is calculated by the derivation of the magnetic energy in air with respect to the rotation angle. The Maxwell stress tensor method is based on the calculation of the magnetic shear traction. Thus the torque can be calculated by integrating magnetic shear traction over a surface enclosing the region. Reichet et al (1976 and 1988) suggested the finite element method (FEM), the volume integral of the force density and the surface integral of Maxwell stress tensor in one single relative position of the movable versus the fixed one to calculate the torque. Coulomb et al (1983 and 1984) has come forward with an elegant scheme for implementing the virtual work principle. It employs the fact that the finite element solutions are functions of space as given by interpolation function, and not just the explicit nodal values. Marinescu et al (1988) and Mizia et al (1988) compared several different methods to calculate the torque.

This paper presents how to calculate and how to characterize the torque in a brushless DC motor. The torque was calculated by the Maxwell stress tensor method, integrating the shear component of magnetic traction along the air gap assuming a quasistatic magnetic field, its characteristics were discussed using spectral analysis. Several different cases are investigated, e.g. the reluctance torque, and commutating torque under center tapped Y-winding, Y-winding and Δ-winding. It will be also discussed.

![Fig. 1] 6-pole, 9-slot and 3-phase motor
<Fig. 2> (a) center tapped Y-winding, (b) Y-winding, (c) Δ-winding
about the proper commutating angle and position of hall effect device.

2. Method of Analysis

The magnetic field generated by a brushless dc motor is governed by the set of Maxwell’s equations. Introducing the scalar potential into the Maxwell equations, with some mathematics, gives a single partial differential equation for the scalar potential.

\[ \nabla^T \mu \nabla \Phi - \nabla^T \mu \mathbf{H} = 0 \]  \hspace{1cm} (1)

The field intensity due to the current in the winding and the permanent magnet, may always be calculated directly by the Biot-Savart law and the magnetic dipole moment per unit volume, respectively. This equation, like the Poisson’s equation for electrostatic fields, can be solved using the finite element method. TOSCA, a FEM solver for magnetic field, was used to calculate the scalar potential \( \Phi \) and the magnetic field intensity, \( \mathbf{H} \).

A non-uniform distributed force per unit area at the interface between two materials is calculated by use of the Maxwell stress tensor. Since the strain imposed on the material due to magnetostriction is small enough to neglect changes in the density, it can be assumed that the change of permeability is negligible. Thus, in tensor notation,

\[ \sigma_{ij} = \frac{1}{\mu} \left( B_i B_j - \frac{1}{2} \delta_{ij} B_k B_k \right) \]  \hspace{1cm} (2)

where \( \sigma_{ij} \) is the Maxwell stress tensor, \( B_i \) is the magnetic flux density which is obtained by the multiplication of permeability to the magnetic field intensity. From the expression given by Woodson and Melcher(1985) for an interface between two materials a and b, the traction, \( f_i \), is given by:

\[ f_i = (\sigma_{ij}^a - \sigma_{ij}^b) n_j \]  \hspace{1cm} (3)

The normal and the tangential traction can be decomposed.

\[ f_n = (\sigma_{ij}^a - \sigma_{ij}^b) n_i n_j \]  \hspace{1cm} (4)

\[ f_t = \sqrt{|A|^2 - |f_n|^2} = |\mathbf{n} \times \mathbf{f} \times \mathbf{n}| \]  \hspace{1cm} (5)

Since \( \mu_{air} \ll \mu_{iron} \), the magnetic traction can be simplified with the introduction of the cylindrical coordinate on the stator. Along the air gap, the normal and the tangential traction for tooth face are as follows:

\[ f_r \approx \sigma_{rr} = \frac{1}{2\mu_{air}} (B_r^2 - B_\theta^2 - B_z^2) \]  \hspace{1cm} (6)

\[ f_\theta \approx \sigma_{r\theta} = \frac{1}{\mu_{air}} B_r B_\theta \]  \hspace{1cm} (7)

But for tooth side which is perpendicular to the air gap, the normal and the tangential traction have the following form:

\[ f_\theta \approx \sigma_{\theta\theta} = \frac{1}{2\mu_{air}} (B_\theta^2 - B_r^2 - B_z^2) \]  \hspace{1cm} (8)

\[ f_r \approx \sigma_{r\theta} = \frac{1}{\mu_{air}} B_r B_\theta \]  \hspace{1cm} (9)

The torque produced for one position can easily be derived from the integration of the shear force along the small air gap with the fact that the field distribution inside a closed surface in air remain unchanged if the external sources are removed and replaced by currents and poles on the surface( Reichert et al 1976 ).

\[ T = \oint R \times f_\theta d\Omega \]  \hspace{1cm} (10)

The magnetic field produced as the rotor rotates, can be thought of as a series of magneto-static fields, or quasi-static magnetic
fields, with the assumption that the magnetic field produced by the permanent magnets and the current is not significantly affected by the other dynamic effects, such as eddy current and the rise and fall time of current.

The magnetic traction acting on a single tooth and the torque can be interpolated and their frequency components can be determined within the accuracy of the Nyquist sampling theorem by multi-dimensional Fourier transform (Newland, 1984). With the period of $L_1$ and $L_2$ corresponding to tooth angle, $\theta$, and rotating angle, $wt$, the magnetic traction is

$$ f(\theta, wt) = \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} Y_{km} e^{j\theta} e^{j\frac{2\pi k}{L_1}} e^{j\frac{2\pi m}{L_2}} $$  \hspace{1cm} (11)$$

The complex Fourier coefficient is defined as

$$ Y_{km} = \frac{1}{L_1 L_2} \int_{0}^{L_1} d\theta \int_{0}^{L_2} dt \cdot f(\theta, wt) e^{-j\frac{2\pi k}{L_1} \frac{t}{L_1}} e^{-j\frac{2\pi m}{L_2} \frac{t}{L_2}} $$  \hspace{1cm} (12)$$

Subscript k and m represent the quantities in the frequency domain corresponding $\theta$ and $wt$. The torque which is obtained by integrating the shear force along the air gap, is

$$ T(wt) = \sum_{n=-\infty}^{\infty} Z_n e^{j\frac{2\pi n}{L_3}} $$  \hspace{1cm} (13)$$

$$ = Z_0 + \sum_{n=1}^{\infty} 2|Z_n| \cos(nwt + \Psi_n) $$  \hspace{1cm} (14)$$

where $Z_0$ and $Z_n$ are the average value and the amplitude corresponding to the frequency component $nw$ and $Y_n = \arg(Z_n)$. The complex Fourier coefficient is defined as

$$ Z_n = \frac{1}{L_3} \int_{0}^{L_3} dt \cdot T(wt) e^{-j\frac{2\pi n}{L_3}} $$  \hspace{1cm} (15)$$

3. Finite Element Modeling

Fig. 3 shows the finite element model for a brushless dc motor with six poles and nine teeth with the tooth slot and permanent magnet slot modeled on the radial line. The permanent magnet has the residual flux density 1.0 T, coercivity -800 kA/m and the parallel magnetization. 3 A/mm² current density is applied to the coil area (50 mm²) for a center tapped Y winding during the commutation. The rotor would rotate in the counterclockwise direction under the given configuration. Permanent magnet slot and the teeth slot correspond to 2 degree, respectively. Air gap between permanent magnet and the teeth is 1 mm. Because the same geometry with the same orientation of the permanent magnets and the current is repeated every 120 degree, one-third of the model is enough to analyze the whole model with periodicity boundary conditions and it has 43,989 nodes.
4. Results and Discussion

Reluctance torque, and commutating torques under center tapped Y-winding, Y-winding and Δ-winding were calculated and their effects were investigated. Table 1 shows the variation of torque at one position as shown in Fig. 3 across the air gap with 1920 integrating points for 120 degree. Even though the same values of torque, as explained in the previous section, are produced across the air gap, the value near the tooth face includes some deviation because there is sharp peak in teeth slot due to a magnetic flux concentration as shown in Fig. 4. This region needs a refined mesh and the quadratic elements to ensure C⁰ continuity of magnetic flux density which decreases the integration error.

The torque values are stationary in the middle of the air gap and the path along 35.4 mm was chosen for torque calculation. Fig. 5 shows the torque diagram for reluctance torque, center tapped Y-winding, Y-winding and Δ-winding. Table 2 shows the frequency components and their amplitudes for each case. It will be explained in detail in the next section.

4.1 Reluctance Torque

Reluctance torque, or cogging torque is produced by the magnetic flux due to the permanent magnet and the tooth geometry only.

It tends to move the rotor to an equilibrium position. Fig. 6 shows the magnetic flux density and the magnetic force in the air gap for the equilibrium position. There is a concentration of magnetic flux near tooth slot. Magnetic shear traction is mainly produced in the tooth slot. Magnetic shear traction has the same distribution with the opposite sign with respect to the center of the tooth slot and its integration is zero in this equilibrium position. Bₚ and Bₚ increased to ±1.143 T and ±0.655 T at both ends of middle tooth so that the magnetic shear traction increased to ±567 KPa. Fig. 5 shows the reluctant torque for 40 degree.

As rotor rotates 20 degree from the equilibrium position as shown in Fig. 3, it has another equilibrium position with different polarity. As rotor rotates 40 degrees, it has an equilibrium position with the same polarity,

<table>
<thead>
<tr>
<th>Radius (mm)</th>
<th>No Current</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CTY-winding</td>
</tr>
<tr>
<td>35.1</td>
<td>-0.01265</td>
<td>25.32059</td>
</tr>
<tr>
<td>35.2</td>
<td>0.04914</td>
<td>25.39366</td>
</tr>
<tr>
<td>35.3</td>
<td>0.01323</td>
<td>25.35805</td>
</tr>
<tr>
<td>35.4</td>
<td>0.00556</td>
<td>25.35139</td>
</tr>
<tr>
<td>35.5</td>
<td>0.00634</td>
<td>25.35213</td>
</tr>
<tr>
<td>35.6</td>
<td>0.00373</td>
<td>25.34991</td>
</tr>
<tr>
<td>35.7</td>
<td>-0.01365</td>
<td>25.33347</td>
</tr>
<tr>
<td>35.8</td>
<td>0.07367</td>
<td>25.41965</td>
</tr>
<tr>
<td>35.9</td>
<td>-0.88489</td>
<td>24.43629</td>
</tr>
</tbody>
</table>
<Fig. 4> Variation of magnetic shear force in the air gap

<Table 2> Frequency components and their amplitudes for torques

<table>
<thead>
<tr>
<th>Number of Frequency</th>
<th>Reluctant Torque</th>
<th>Commutating Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CTY-winding</td>
</tr>
<tr>
<td>0</td>
<td>-0.01265</td>
<td>25.38</td>
</tr>
<tr>
<td>18</td>
<td>0.04914</td>
<td>0.41</td>
</tr>
<tr>
<td>36</td>
<td>0.01323</td>
<td>0.38</td>
</tr>
<tr>
<td>54</td>
<td>0.00556</td>
<td>0.17</td>
</tr>
<tr>
<td>72</td>
<td>0.00634</td>
<td>0.07</td>
</tr>
<tr>
<td>90</td>
<td>0.00373</td>
<td>0.01</td>
</tr>
</tbody>
</table>
<Fig. 5> Torque diagram for reluctant torque, center tapped Y-winding, Y-winding and Δ-winding which is the same relative position of rotor with respect to the stator. The torque diagram, however, repeats every 20 degrees because torque is produced by the integration of the shear force, which is governed by the product of $B_r$ and $B_0$ and their sign changes together after 20 degrees of rotation. This explains the driving frequency of the reluctant torque at the 18th, 36th, 54th harmonics and so on, which are integer multiples of the least common multiple of the num-ber of pole and the number of teeth as shown in Fig. 7. Frequency components and their amplitudes for the reluctance torque are given in Table 2. The average value of the reluctance torque is negligible and the amplitude decreases as driving frequency increases.

### 4.2 Commutating Torque under Center Tapped Y-winding

Commutating torque is produced by the interaction between the permanent magnet and the commutating coil current. The current commutates in such a way to rotate a rotor in
The commutating angle is determined by 360 degrees divided by the product of pole number and phase number. Fig. 8 shows the magnetic flux density and the magnetic traction in the air gap for a motor with 6 poles, 9 teeth and 3 phase center tapped Y-winding as shown in Fig. 2(a). The current, 3 A/mm² is applied to the middle tooth B with coil area 50 mm². The current increases the magnetic flux at the end of tooth in the rotating direction and decreases it in the opposite direction in order to produce an unbalanced magnetic shear traction. The commutating torque is the integration of this unbalanced
<Fig. 7> Reluctance torque and its frequency spectrum

magnetic traction along the air gap. In Fig. 8, \( B_r \) and \( B_0 \) decreased to 0.996 T and 0.51 T in the left end of middle tooth, B, so that the magnetic shear traction decreased to 378.48 KPa. In the right end of middle tooth, B, and \( B_0 \) increased to -1.28 T and 0.805 T so that the magnetic shear traction increased to -759.99 KPa.

Fig. 5 shows the commutating torque for a 40 degree range. Even though it has wide commutating range from -19 degree to +19 degree for center tapped Y-winding, the maximum torque can be obtained by energizing the central 20 degrees. For this commutation, a hall effect switch can be located 10 degree ahead from the center of tooth. After the rotor rotates 20 degree from -10 to 10 degree, the coil in the next left tooth, A, is energized with a different current direction and the magnetic flux pattern facing it has a different direction. The commutating torque thus repeats every 20 degree. This explains the driving frequencies of the commutating torque the 18th, 36th, 54th harmonic, and so on, which are integer multiples of the pole number and the number of phase. Fig. 9 shows the frequency spectrum of center tapped Y-winding. This frequency spectrum is similar to that of the reluctance torque in Fig. 7 except for the average value. Table 2 shows the frequency components and their amplitudes. This explains that the one of the major source of torque ripple is the reluctance torque.

The motor equations will now be discussed to compare the characteristics of each windings for the motor with 6 poles, 9 teeth and 3 phase. If we assume that the magnetic flux density has an average value of, \( B_r \) the torque, \( rB_i l \) should work on every conductor where \( r \), \( i \) and \( l \) are radius, current and axial thickness of motor. Considering that the both ends of coil of the middle tooth for 1/3 model are active, the whole torque for N turns of coil is

\[
T = k_c i \text{ where } k_c = \frac{6NrB_r l}{(16)}
\]

Because the torque constant and the back e.m.f. constant are same in SI units, the voltage equation is

\[
V = R_c i + k_c w \text{ (17)}
\]

where \( R_c = 3R \) and R is the resistance of coil around a tooth. The electric power supplied from the power source is

\[
V_i = R_c i^2 + k_c w i = R_c i^2 + Tw \text{ (18)}
\]
where $R_f^2$ is the heat loss and $T$ is the mechanical output power.

4.3 Commutating Torque under Y-winding

Fig. 10 shows the magnetic flux density and the magnetic traction in the air gap for the motor with 6 poles, 9 teeth and 3 phase Y-winding shown in Fig. 2(b).

The current, 1.5 A/mm², is applied to teeth A and B with the coil area of 50 mm² respectively to produce the equivalent of torque, because the torque constant of Y-winding is twice bigger than that of center tapped Y-winding. Because the same level of magnetic flux would exist for a closed loop of magnetic flux along teeth B and A due to the current in coil B and
A, compared with the center tapped Y-winding, the same values $B_r$ and $B_o$ and magnetic shear traction at the left end of tooth B are expected. These are 1.01 T, 0.53 T and 400.1 KPa for the left end of tooth face. However, because the level of magnetic flux would be half for the closed loop of magnetic flux along the teeth B and C, half the increase of $B_r$, $B_o$ and magnetic shear traction at the right end of tooth B is expected, which are -1.23 T, 0.74 T and -671.1 KPa for the right end of the tooth face.

Fig. 5 shows the commutating torque for a 40 degree range. A Y-winding has narrow commutation range: 20 degrees just after the equilibrium position under no current condition. For this commutation, hall effect device is located at the center of tooth. After rotor rotates 20 degrees (from 0 to 20 degree), the coils in B and C are energized with different current directions and the magnetic flux pattern facing it has different direction, so that commutating torque repeats every 20 degree. This explains that the driving frequency of the commutating torque is at the 18th, 36th, 54th harmonics, and so on, which are the integer multiple of the pole number and the phase number.

Fig. 11 shows the frequency spectrum of Y-winding. Because Y-winding has a narrow commutation range, the average value is smaller than that of the center tapped Y-winding and the amplitudes of driving frequencies are higher than those of the center tapped Y-winding, as shown in Table 2.

The Y-winding has a different torque constant and different total resistance. Because the both ends of coils in teeth A and B are active for 1/3 model, the torque constant is twice that of a center-tapped Y-winding.

$$k_Y = 12N\sigma B_r l$$

(19)

Hence, half the current should be applied in order to produce the same amount of torque as a center-tapped Y-winding.

$$i_Y = \frac{1}{2} i$$

(20)

The total resistance is $R_Y = 6R$ because the resistance is 2R for the 1/3 model as shown in Fig. 2(b). However, the heat loss is half compared with the center-tapped Y-winding because the amount of current is half.

$$i_Y^2 R_Y = \frac{1}{2} i^2 R_c$$

(21)
4.4 Commutating Torque under Δ-winding

Fig. 12 shows the magnetic flux density and the magnetic traction in the air gap for the motor with 6 poles, 9 teeth and 3 phase Δ-winding. The current, 2 A/mm² to tooth B, 1.5 A/mm² to teeth A and C are applied to produce the equivalent torque because two third of current flows in B and each one third of current flows in A and C, respectively with the given directions as shown in Fig. 2(c). The magnetic flux density along the air gap is exactly the same as that of center tapped Y-winding, as shown in Fig. 8. Because the same level of magnetic flux would exist.
for a closed loop of magnetic flux along the teeth B and A due to the current in coil B and A and for the closed loop of magnetic flux along the teeth B and C due to the current in coil B and C, compared with the center-tapped Y-winding. The magnetic flux from tooth A to the tooth on the left of tooth A due to current compensates the magnetic flux from the tooth on the left A to tooth A because the directions of magnetic flux are opposite. Thus the magnetic flux due to current is zero in that area. Hence the magnetic flux density around tooth A is exactly the same as that of center-tapped Y-winding, and the magnetic flux around tooth C is also exactly the same as that of center-tapped Y-winding.

Fig. 5 shows the commutating torque for 40 degree. Torque curves of the \( \Delta \)-winding is exactly the same as that of the center tapped Y-winding because the role of current is exactly the same as that of center tapped Y-winding. Hence, it produces the same frequency spectrum as shown in Fig. 9. The location of hall effect device is also 10 degree ahead from the center of tooth, like the center-tapped Y-winding.

\( \Delta \)-winding has the same torque constant as center tapped Y-winding but the different total resistance. The active coils are in the slot between tooth A and B, and the slot between B and C, which is the exactly the same as center tapped Y-winding.

\[
k_\Delta = 6N_{r}B_{r}l
\]  

(22)

Hence, the same amount of current which is distributed 2/3 for one tooth and 1/3 for other two teeth, should be applied in order to produce the same amount of torque.

\[
i_\Delta = i
\]  

(23)

The total resistance is \( R_\Delta = 2R \) because the resistance is 2/3 R for 1/3 model as shown in Fig. 2(c). Hence, the heat loss is the 2/3 compared with the center tapped Y-winding.

\[
i^2_\Delta R_\Delta = \frac{2}{3} i^2 R_c
\]  

(24)

4.5 Torque Ripple

Torque ripple is the difference between the maximum and the minimum torque. Even though one of major source of torque ripple in brushless DC motor is due to switching action of current, reluctance torque is becoming the another major source as the advent of rare earth magnet. At high speeds, torque ripple is
usually filtered out by the system inertia. However, at low speeds, torque ripple produces noticeable effects that may not be tolerable.

The switching action of current is dependent on the phase feeding of current, PWM frequency, its duty cycle and so on. But for the same number of phases, the winding scheme does not make any effect to the torque ripple because different windings with same number of phase does not change the electrical time constant which is the ratio of armature inductance to armature resistance and determines the rising time of current. The Y-winding increases the resistance and the inductance twice.
together, and the \( \Delta \)-winding decreases them by two-third.

5. Conclusion

The reluctance torque is produced by the permanent magnet and the teeth geometry, increasing the magnetic flux concentration in the teeth slot with the tendency to move the rotor to an equilibrium position. The driving frequency of the reluctance torque is determined by the integer multiples of the least common multiple of the number of pole and the number of teeth.

The commutating torque is produced by the interaction of the permanent magnet and the current which increases the magnetic field concentration of the teeth corner in the moving direction and decreases it in the opposite direction. The driving frequency of the commutating torque is defined by the integer multiple of the number of pole and the number of phases.

Y-winding reduces the winding heat loss by half, compared with center tapped Y-winding, but it has narrow commutating range and hall effect device is located 10 degree ahead from the center of teeth. It produces high amplitudes of driving frequencies. Center tapped Y-winding and \( \Delta \)-winding have the wide commutating range, and the hall effect device is located at the center of teeth. \( \Delta \)-winding produces the same torque as center tapped Y-winding, but it reduces the winding heat loss by two thirds.

References